

() PROBLEME DE CYCLAGE : EXEMPLE DE BEALE**

Soit le programme linéaire :

$$\begin{array}{l} \text{Maximiser} \\ z = 3/4 \, x_1 - 150x_2 + 1/50 \, x_3 - 6x_4 \\ \text{avec} \\ 1/4 \, x_1 - 60x_2 - 1/25 \, x_3 + 9x_4 \leq 0 \\ 1/2 \, x_1 - 90x_2 - 1/50 \, x_3 + 3x_4 \leq 0 \\ x_3 \leq 1 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0. \end{array}$$

1. Montrer que la méthode du simplexe conduit à un cyclage, lorsqu'en cas d'égalité pour le 2^e critère de Dantzig, on fait sortir de la base la variable dont la ligne est la plus haute dans le tableau.
2. Utiliser la règle de Bland pour trouver une solution optimale.
3. Que se passe-t-il si l'on change le signe du profit de la variable x_2 dans la fonction économique ?

CORRIGÉ

1. Pour simplifier la présentation, nous ne ferons pas apparaître dans les différents tableaux du simplexe la première colonne des coefficients, dans la fonction économique des variables de base ni la ligne des coefficients de cette fonction.

Les différents tableaux du simplexe sont les suivants :

The diagram illustrates a sequence of operations on a 3x3 matrix. It begins with a matrix i and a vector s . The matrix i is shown as a box with rows $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$. The vector s is shown as a box with rows $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. This is followed by a series of operations involving matrices u , e , and f , and vectors z and j . The matrices are represented as boxes with numbers inside, and the vectors as boxes with numbers inside. The operations are indicated by arrows and labels like Δ_j and Δ_f .

The sequence of operations is as follows:

- Matrix i and vector s are combined to form a new matrix (shown as a box with rows $\begin{bmatrix} 1/4 & -60 & -1/25 \\ 1/2 & -90 & -1/50 \\ 0 & 0 & 1 \end{bmatrix}$).
- This matrix is then transformed by Δ_j to form a new matrix (shown as a box with rows $\begin{bmatrix} 3/4 & -150 & 1/50 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$).
- This matrix is then transformed by Δ_f to form a new matrix (shown as a box with rows $\begin{bmatrix} 1 & -240 & -4/25 \\ 0 & 30 & 3/50 \\ 0 & 0 & 1 \end{bmatrix}$).
- This matrix is then transformed by Δ_j to form a new matrix (shown as a box with rows $\begin{bmatrix} 0 & 30 & 7/50 \\ 0 & -33 & -3 \\ 0 & 0 & 0 \end{bmatrix}$).

Figure 10 displays a series of 12 matrices and vectors, arranged in a grid, showing the steps of the simplex method. The matrices are labeled with 's' and 'i' for rows and columns, and 'Δj' for the objective function row. The vectors are labeled 'u' and 'z'.

The matrices are arranged in a 4x3 grid:

- Row 1: Matrix 1, Matrix 2, Matrix 3
- Row 2: Matrix 4, Matrix 5, Matrix 6
- Row 3: Matrix 7, Matrix 8, Matrix 9
- Row 4: Matrix 10, Matrix 11, Matrix 12

The vectors are arranged in a 4x3 grid:

- Row 1: Vector 1, Vector 2, Vector 3
- Row 2: Vector 4, Vector 5, Vector 6
- Row 3: Vector 7, Vector 8, Vector 9
- Row 4: Vector 10, Vector 11, Vector 12

The matrices show the progression of the simplex method, including the selection of the pivot element and the resulting tableau.